12 – Math and Physics Review: Pre-(College) Algebra Topics: Set Notation

Complex Number Set

$$\mathbb{C} = \left\{a + ib \middle| a, b \in \mathbb{R}, i = \sqrt{-1}\right\}, \qquad \vec{z} = \langle a, b\rangle, \qquad \|\mathbf{z}\| \equiv |\vec{z}| = \sqrt{a^2 + b^2}$$

Real Number Set, $\mathbb{R} \subset \mathbb{C}$

$$\mathbb{R} = (-\infty, \infty) \text{ all real numbers } \mathbb{C} = \left\{a + i(0) \middle| a, b = 0 \in \mathbb{R}, \cancel{i = \sqrt{-1}}\right\} \equiv \mathbb{R}$$

- Rational Numbers, $\mathbb{Q} \subset \mathbb{R}$

$$\mathbb{Q} = \left\{\frac{a}{b} \middle| a, b \in \mathbb{Z}, b \neq 0\right\}$$

- All Integers, $\mathbb{Z} \subset \mathbb{Q}$

$$\mathbb{Z} = \{\dots - 3, -2, -1, 0, 1, 2, 3, \dots\}, \qquad \mathbb{Q} = \left\{\frac{a}{b} \middle| a, b = 1 \in \mathbb{Z}, b \neq 0\right\} \equiv \mathbb{Z}$$

- Positive Integers (natural numbers), $\mathbb{N} \equiv \mathbb{Z}^+ \subset \mathbb{Z}$

$$\mathbb{Z}^+ = \mathbb{N} = \{1, 2, 3, \dots\}, \qquad \mathbb{Z}^+ \cup \{0\} = \mathbb{N} \cup \{0\} = \{0, 1, 2, 3, \dots\}$$

- Irrational Set, $\mathbb{R} \setminus \mathbb{Q}$

$$\mathbb{I}^* \equiv \mathbb{R} \setminus \mathbb{Q} = \left\{e, \pi, \sqrt{2}, \dots\right\}$$

Universal Quantifiers

- in, $\in$
- not in, $\notin$
- such that, $\ni$
- therefore, $\therefore$
- because, $\because$
- exist, $\exists$
- implies, $\Rightarrow$
- if and only if (iff), $\Leftrightarrow$
- modulo or Equivalent, $\equiv$
- goes to, $\rightarrow$
- subset, $\subset$
- proper Subset $\subseteq$
- for all, $\forall$
- divides, $|$
- does not divide, $\nmid$
- tilda, $\sim$
- and, $\wedge$
- or, $\vee$

Complex Numbers in Set Notations and Graphing

Complex Number Set

$$\mathbb{C} = \left\{a + ib \middle| a, b \in \mathbb{R}, i = \sqrt{-1}\right\}, \qquad \vec{z} = \langle a, b\rangle, \qquad |\vec{z}| = \sqrt{a^2 + b^2} = \|\mathbf{z}\|$$

Note a few things in relation to complex numbers:

$$z = x + iy = \mathrm{Re}\{z\} + \sqrt{-1}\mathrm{Im}\{z\}, \qquad x = a + ib = \alpha + i\beta$$

$$\left\{a + ib \middle| a, b \in \mathbb{R}, i = \sqrt{-1}\right\} \equiv \left\{0, 1, 1 + i, i, \dots, 1 + 2i, \sqrt{2} + \sqrt{5}i, \dots\right\}$$

Now, for vector operations such as the dot product:

$$z = x + iy = \langle x, y\rangle \cdot \langle 1, i\rangle = \begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}^T \begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 \\ i \end{bmatrix} = (x)(1) + (y)(i).$$

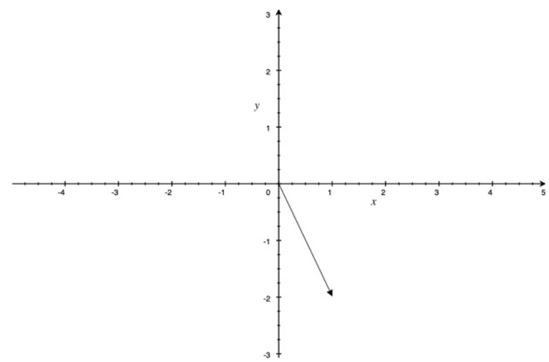
$$\mathbf{z} = \vec{z} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Consider, $z = 1 - 2i \;\Rightarrow\; \vec{z} = \langle 1, -2\rangle$.

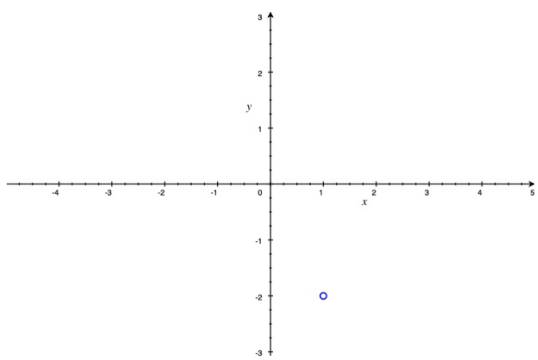
The following three graphs represent the bulk of the extracted point, vector, and length of the vector.

$$\vec{z} = \langle 1, -2\rangle,$$
$$Z(1, -2),$$
$$|\vec{z}| = \sqrt{(1)^2 + (-2)^2} = \sqrt{5}.$$

$\vec{z} = \langle 1, -2\rangle$



$Z_0(1, -2)$



$|\vec{z}| = \sqrt{(1)^2 + (-2)^2} = \sqrt{5}$