3 – Before Starting Math as a STEM Major

Mathematics is a language used to describe phenomena and relay information. It is not magic, and it is not physics—it is language. ChatGPT Rewrite of math statement: (original follows)

"**Introduction**

In the study of calculus, one of the most powerful tools for evaluating integrals is the substitution method, often referred to in slang as "U-Sub." This method transforms an otherwise difficult integral into a simpler one by means of a clever change of variables. In this essay, we shall examine a definite integral that combines trigonometric and exponential functions, and we shall demonstrate how substitution allows us to evaluate it step by step.

Body of the Argument

We begin with the integral from zero to pi divided by two of sine of theta multiplied by e raised to the cosine of theta, with respect to theta.

That is, the integral of sine theta times e to the cosine theta, d theta, from zero to pi over two.

The substitution formula states:
The integral from a to b of f of g of x, multiplied by g prime of x, with respect to x, is equal to the integral from g of a to g of b of f of u with respect to u, which is equal to big F of g of b minus big F of g of a.

Let us now apply this principle.

We let u equal cosine of theta. Then, the differential du is equal to negative sine theta d theta. From this, it follows that negative du is equal to sine theta d theta.

At the lower limit, when theta equals zero, u equals cosine of zero, which is one. At the upper limit, when theta equals pi over two, u equals cosine of pi over two, which is zero.

Thus, the original integral becomes the integral from one to zero of e to the power of u multiplied by negative du. Reversing the sign, this becomes the integral from zero to one of e to the u, du.

Evaluating this gives e to the u from zero to one, which equals e to the first power minus e to the zero power. This is e minus one.

Additional Note

One should remember that the integral from a to b of dx is equal to negative the integral from b to a of dx, and similarly, the negative integral from one to zero of e to the u du equals the integral from zero to one of e to the u du. These facts justify the reversal of limits in the step above.

Summary Therefore, the definite integral from zero to pi divided by two of sine of theta multiplied by e raised to the cosine of theta, with respect to theta, is equal to e minus one.

This example illustrates the efficiency of the substitution method in transforming a trigonometric–exponential integral into a simple exponential integral. By a clear substitution and proper adjustment of limits, we find that a seemingly complex integral reduces to the simple difference e minus one." **Original Solution** (written symbolically):

[0 – Title of Essay: Question] **Substitution – Trigonometry – Exponential – DEFINITE**

$$\int_0^{\frac{\pi}{2}} \sin\theta\, e^{\cos(\theta)}\, d\theta = -\int_0^{\frac{\pi}{2}} -\sin\theta\, e^{\cos(\theta)}\, d\theta$$

[1 – Formulae | Header and Citation of the Essay] 50% of the Credit

Clearly, this is the Substitution Method (often referred to in slang as U-Sub).

$$\int_a^b f\big(g(x)\big)\cdot g'(x)\,dx = \int_{u_1=g(a)}^{u_2=g(b)} f(u)\,du = F\big(g(b)\big) - F\big(g(a)\big)$$

[2 – Formulate | Body of the Essay] Lose up to 50% of Credit $u = \cos\theta \;\Rightarrow\; du = -\sin\theta\, d\theta \;\Rightarrow\; -du = \sin\theta\, d\theta$

$$u(0) = \cos(0) = 1, \qquad u\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$$

$$\int_0^{\frac{\pi}{2}} \sin\theta\, e^{\cos(\theta)}\, d\theta = \int_0^{\frac{\pi}{2}} e^{\cos(\theta)} \sin\theta\, d\theta = \int_1^0 e^u\,(-du) = -\int_1^0 e^u\,du =$$

$$-[e^u]_1^0 = -(e^0 - e^1) = e - e^0 = e - e^{1-1} = e - e^1 e^{-1} = e - \frac{e^1}{e^1} = e - 1$$

NOTE: $x^0 = 1$ as shown above. And

$$\int_a^b dx = -\int_b^a dx\,, \qquad -\int_1^0 e^u\,du = \int_0^1 e^u.$$

[3 – Finalize | Summarization of the Essay] 0% of Credit (but you can lose points still)

$$\therefore \int_0^{\frac{\pi}{2}} \sin\theta\, e^{\cos(\theta)}\, d\theta = e - 1.$$

