Integral Calculus

Before Starting Antiderivatives

Before you begin integration, understand that it is not really math; rather, it is puzzle solving and arithmetic. And, as with all puzzles, there is a process of elimination that must be followed to solve them effectively and efficiently on exams.

The following is an example of how I process integration in my head before finalizing the approach.

Zeroth Step: Definite or Indefinite?

$$\int f(x)\,dx \text{ [indefinite]}, \qquad \int_a^b f(x)\,dx \text{ [definite]}.$$

[0.1] If **definite**: Is the integrand defined on the interval? I.e., is $f(x)$ continuous for all x in the interval. If the boundaries are infinite or the function is not defined, then it is an improper integral.

[0.2] If $x \in [-a, a]$, is $f(x)$ even, odd, or neither?

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 0, & f(-x) = -f(x) \text{ is odd} \\ 2\int_0^a f(x)\,dx, & f(-x) = f(x) \text{is even} \end{cases}.$$

[0.3] If improper, you have Type I (discontinuous/undefined) and Type II (infinite)

$$\int_0^1 \frac{1}{x}\,dx = \lim_{t\to 0^-} \int_t^1 \frac{1}{x}\,dx = \text{Diverges}, \qquad x \neq 0, \qquad \int_0^\infty -2xe^{-x^2}\,dx = -1 = \text{Converges}.$$

NOTE: The symbol $\int$ is the letter "S" short for summation—i.e., Σ. $f(x)$ is the integrand and dx is the differential or infinitesimally small change in x.

$$[\text{S}] \int f(x)[\text{integrand}]\; dx[\text{differential}]$$

[Step 1] Can I use an existing formula to integrate?
[Step 2] Can I manipulate the integrand so I can use a formula?
[Step 3] Can I use the substitution method?
[Step 4] Can I use trigonometric substitution?
[Step 5] Do I need to use partial fractions or long(synthetic) division?

[Step 6] Can I use Integration by Parts?
[Step 7] Can I use a complex number relation such as $e^{i\theta} = \cos\theta + i\sin\theta$?

Steps 2 through 7 (and more) are interchangeable. The general idea is that you are using the process of elimination to pinpoint the only possible possibilities. I call it having "The Eye of the Integral." Or the Sherlock Holmes method of problem solving—to eliminate the impossible, leaving left, however impossible it seems, the only possible possibility.

