Integrating the Dirac Delta Function over all Space

[12] 15.(c) Use (11.6) and (11.14) to (11.16) to evaluate the following integrals. Warning hint: See comments just after (11.6) and (11.16) about the range of integration.

Dirac Delta Function

$$\int_{\mathbb{R}} \phi(x)\delta^{(n)}(x-a)\,dx = (-1)^n \phi^{(n)}(a)$$

Evaluate the following integral,

$$\int_{-1}^{1} e^{3x}\delta'(x)\,dx.$$

First get it to match the formula.

$$\int_{-1}^{1} e^{3x}\delta'(x)\,dx = \int_{-1}^{1} e^{3x}\delta^{(1)}(x-0)\,dx$$

For this situation, we are not integrating over all space. So we look to another definition or theorem.

The boundaries for

$$\int_{-\infty}^{\infty} \phi(x)\delta^{(n)}(x-a)\,dx$$

$$= \int_{-\infty}^{-1} \phi(x)\delta^{(n)}(x-a)\,dx + \int_{-1}^{1} \phi(x)\delta^{(n)}(x-a)\,dx + \int_{1}^{\infty} \phi(x)\delta^{(n)}(x-a)\,dx$$

$$= 0 + \int_{-1}^{1} \phi(x)\delta^{(n)}(x-a)\,dx + 0$$

$$= \int_{-1}^{1} e^{3x}\delta^{(1)}(x-0)\,dx = (-1)^1 \frac{d}{dx} e^{3x}\bigg|_{x=1} = -3e^{3x}.$$

Is this conclusion correct? The question told us to use (11.6) and (11.14) to (11.16). Now, you can go to the book and think and come to a conclusion of whether or not this is correct after reading.

I will come back to this after you all review it and put your input in.