Thermodynamics: [17.4] Thermal Expansion

[3] 17.22 A brass rod 185 cm long and 1.60 cm in diameter. What **force** must be applied to each end of the **rod** to prevent it from **contracting** when it is **cooled** from 120.0°C to 10.0°C?

- Identify Significant Figure: The least sig fig count is 3
- Identify **Keywords: contracting, cooled, rod, force**

[1st – Extract Data] 50% of the credit on exams.

Given (directly give)	Need (indirectly give)	Seeking
$L_0 = 185$ cm, $d = 1.60$ cm	$\Delta L = \alpha L_0 \Delta T$	$F = ?$
$T_0 = 120.0°C$, $T = 10.0°C$	$\dfrac{F}{A} = -T\alpha\Delta T$	$\dfrac{F}{A} = -Y\alpha\Delta T$
$\Delta T = 10.0°C - 120.0°C = -110.0°C$	$\Delta T = -110.0°C = -110.0$ K	$\Rightarrow F = -AY\alpha\Delta T$
$A = \pi r^2 = \pi\left(\dfrac{d}{2}\right)^2 = \pi\left(\dfrac{1.60\text{ cm}}{2}\right)^2$		
$\alpha = 2.0 \times 10^{-5}\text{K}^{-1} = 2.0 \times 10^{-5}(\text{C}°)^{-1}$		
$Y_{\text{Brass}} = 9.0 \times 10^{10}$ Pa		

NOTE The *interval* in Kelvin is the same as Celsius

Initial Length / Final Length

$$\alpha \text{ units are K}^{-1} \text{ or } (\text{C}°)^{-1}$$

From [3], "If an object has length L_0 at temperature T_0, then its length L at temperature $T = T_0 + \Delta T$ is

$$L = L_0 + \Delta L = L_0 + \alpha L_0 \Delta T = L_0(1 + \alpha\Delta T) \quad [17.7]$$

Thermal Stress

$$\frac{F}{A} = -Y\alpha\Delta T$$

- F, is the force needed to keep the length of the rod constant.

- A, Cross sectional area
- Y, Young's modulus [for brass 9.0×10^{10}]
- α, Coefficient of linear expansion
- ΔT, Temperature change

[2ⁿᵈ – Formulate Data] 50% of the credit on exams.

Now, in theory, we have

$$F = -AY\alpha\Delta T$$

$$= -\left(\pi\left(\frac{1.60\text{ cm}}{2}\cdot\frac{1\text{ m}}{100\text{ cm}}\right)^2\right)[9.0\times10^{10}\text{ Pa}](2.0\times10^{-5}\text{K}^{-1})(-110.0)\text{K}$$

$$= -\left(\pi\left(\frac{1.60}{200}\text{ m}\right)^2\right)[9.0\times10^{10}\text{ Pa}](2.0\times10^{-5}\text{K}^{-1})(-110.0)\text{K} = 39810.3\text{ N}.$$

[3ʳᵈ – State the Answer] 0% of the credit on exams (but no credit without it).

Thus,
$$F = 3.98 \times 10^4\text{ N}.$$

[FC] We do not have an answer for this in the book. How do we verify the work is correct without using a solution manual or A.I.

$$F = -AY\alpha\Delta T \;\Rightarrow\; \Delta T = -\frac{F}{AY\alpha}$$

Of course, the above, will work to verify since it is the same numbers used to find F so it is not going to satisfy our full circle solution. We would need another equation that represents a given measurement with a different measurement that incorporates the found measurement. If the measurement not involved with finding the force comes up correct, then we can be confident our work is correct.

At this point in the question, my concern is with the temperature logic. The book says the difference of temperature is the same in Celsius as it is Kelvin but our change is negative and Kelvin is greater than zero.

I request A.I.(ChatGPT) to provide an answer and request that it does not provide a solution so I can simply compare: ChatGPT says 3.8×10^4 N, however, our answer is 3.98×10^4 N. Is this good enough? NO!!! We must be certain. It is important to stick with your textbooks measurements as the precision of measurements changes over time and AI is not necessarily

drawing from the current or past data—it is not acceptable for accuracy but can help us in our quest.

Continuing the full circle. We should now, consider a different equation that incorporates a constant and our measures that will output a given measurement with our calculated measurement and then we can feel (more) confident that our answer is in fact correct.

IMPORTANT I did not read this section/chapter before attempting this problem, so I am missing key information that points me to the correct solution. One of the problems we encountered was Young's Modulus. I assumed it was universal on all material, and I chose Aluminum instead brass.

HOMEWORK For the student, your job is to find a different formula that you can use to verify the answer is correct. Hint: Possibly a density formula...?

STOP-CONTINUE for Research Position Student's

We will come back to this and find a way to prove whether or not the answer is correct in the next lesson.