Taylor and Maclaurin Series

[7] Use the Taylor series generated by e^x at $x = a$ to show that

$$e^x = e^a \left[1 + (x-a) + \frac{(x-a)^2}{2!} + \cdots \right].$$

Taylor Series

$$T_n(x) = \sum_{n=0}^{n} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

This is a very basic question. Simply apply the formula.

Let $f(x) = e^x$. Then, $f'(x) = e^x, f''(x) = e^x, \ldots$

Now, expand the series out to $n = 1$.

$$\sum_{n=0}^{n} \frac{f^{(n)}(a)}{n!}(x-a)^n = \frac{f^{(0)}(a)}{0!}(x-a)^0 + \sum_{n=1}^{n} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Recall the set of nth order derivatives, $\{y, y', y''', \ldots\} \equiv \left\{y^{(0)}, y^{(1)}, y^{(2)}, \ldots, y^{(n)}\right\}$.

I.e., the 'zeroth derivative' is simply the non-derivative function or antiderivative of y', provided the constant is zero.

$$= \frac{f^{(0)}(a)}{0!}(x-a)^0 + \sum_{n=1}^{n} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$= \frac{f(a)}{0!}(x-a)^{1-1} + \sum_{n=1}^{n} \frac{f^{(n)}(x-a)}{n!}(x-a)^n$$

$$= \frac{f(a)}{0!}(x-a)^1(x-a)^{-1} + \sum_{n=1}^{n} \frac{f^{(n)}(x-a)}{n!}(x-a)^n$$

$$= \frac{e^a}{1}\frac{(x-a)}{(x-a)} + \sum_{n=1}^{n} \frac{f^{(n)}(x-a)}{n!}(x-a)^n = e^a + \sum_{n=1}^{n} \frac{f^{(n)}(x-a)}{n!}(x-a)^n$$

You can now complete this on your own, and we will move on to something else.